

Où est passé le ${}^7\text{Li}$ produit aux premiers instants de l'Univers ?

Tableau périodique des éléments

1 (1s)
18

	1 IA																2 VIII A	
1	1 H 1,008 Hydrogène	2 Be 9,01 Béryllium																10 Ne 20,18 Néon
2		4 Mg 24,31 Magnésium																18 Ar 39,95 Argon
3	11 Na 22,99 Sodium	12 Mg 24,31 Magnésium																36 Kr 83,80 Krypton
4	19 K 39,10 Potassium	20 Ca 40,08 Calcium	21 Sc 44,96 Scandium	22 Ti 47,87 Titane	23 V 50,94 Vanadium	24 Cr 52,00 Chrome	25 Mn 54,94 Manganèse	26 Fe 55,85 Fer	27 Co 58,93 Cobalt	28 Ni 58,69 Nickel	29 Cu 63,55 Cuivre	30 Zn 65,41 Zinc	31 Ga 69,72 Gallium	32 Ge 72,64 Germanium	33 As 74,92 Arsenic	34 Se 78,96 Sélénium	35 Br 79,90 Brome	36 Kr 83,80 Krypton
5	37 Rb 85,47 Rubidium	38 Sr 87,62 Strontium	39 Y 88,91 Yttrium	40 Zr 91,22 Zirconium	41 Nb 92,91 Niobium	42 Mo 95,94 Molybdène	43 Tc [98] Technétium	44 Ru 101,07 Ruthénium	45 Rh 106,42 Rhodium	46 Pd 106,42 Palladium	47 Ag 107,87 Argent	48 Cd 112,41 Cadmium	49 In 114,82 Indium	50 Sn 118,71 Étain	51 Sb 121,76 Antimoine	52 Te 127,60 Tellure	53 I 126,90 Iode	54 Xe 131,29 Xénon
6	55 Cs 132,91 Césium	56 Ba 137,33 Lanthane	57 La 138,91 Lanthane	72 Hf 178,49 Hafnium	73 Ta 180,95 Tantale	74 W 183,84 Tungstène	75 Re 186,21 Rhenium	76 Os 190,23 Osmium	77 Ir 192,22 Iridium	78 Pt 195,08 Platine	79 Au 196,97 Or	80 Hg 200,59 Mercure	81 Tl 204,38 Thallium	82 Pb 207,2 Plomb	83 Bi 208,98 Bismuth	84 Po [209] Polonium	85 At [210] Astaté	86 Rn [222] Radon
7	87 Fr [223] Francium	88 Ra [226] Radium	89 Ac [227] Actinium	104 Rf [261] Rutherfordium	105 Db [262] Dubnium	106 Sg [266] Seaborgium	107 Bh [264] Bohrium	108 Hs [277] Hassium	109 Mt [268] Meitnerium	110 Ds [281] Darmstadtium	111 Rg [...] Roentgenium	...						
				58 Ce 140,12 Cérium	59 Pr 140,91 Prasodyme	60 Nd 144,24 Néodyme	61 Pm [145] Prométhium	62 Sm 150,36 Samarium	63 Eu 151,96 Europium	64 Gd 157,25 Gadolinium	65 Tb 158,93 Terbium	66 Dy 162,50 Dysprosium	67 Ho 164,93 Holmium	68 Er 167,26 Erbium	69 Tm 168,93 Thulium	70 Yb 173,04 Ytterbium	71 Lu 174,97 Lutétium	
				90 Th 232,04 Thorium	91 Pa 231,04 Protactinium	92 U 238,03 Uranium	93 Np [237] Neptunium	94 Pu [244] Plutonium	95 Am [243] Américium	96 Cm [247] Curium	97 Bk [247] Berkélium	98 Cf [251] Californium	99 Es [252] Einsteinium	100 Fm [257] Fermium	101 Md [258] Mendélévium	102 No [259] Nobélium	103 Lr [262] Lawrencium	

Numéro atomique : 6 **C** **Symbole de l'élément**

Principaux nombres d'oxydation : (le plus fréquent est en gras)
+4, +2, +3, +6

Masse atomique
12,01

Electronégativité (Pauling)
2,5

Nom : Carbone

(2s) : deux électrons célibataires
(3p) : trois paires d'électrons

métaux	métaux de transition	non métaux	gaz rares et inertes
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éléments artificiels

L'astérisque (*) signifie élément radioactif (instable)

Les masses atomiques actualisées (2001) sont tirées du site IUPAC : <http://www.chem.qmc.ac.uk/iupac/AW/>

© IUPAC 2005

(*) Les coefficients d'électronégativité sont ceux de Mulliken (J.Chem.Ed. 65 (1968) p. 34) complétés pour les métaux de transition, par ceux du livre de L.S.P. Pauling (Chemistry, W.H.Freeman & Co (1975) p.175)



LAFTH

Pasquale Dario Serpico (Annecy-le-Vieux)
Journées SFP-BTPN - Paris, 21/06/2016



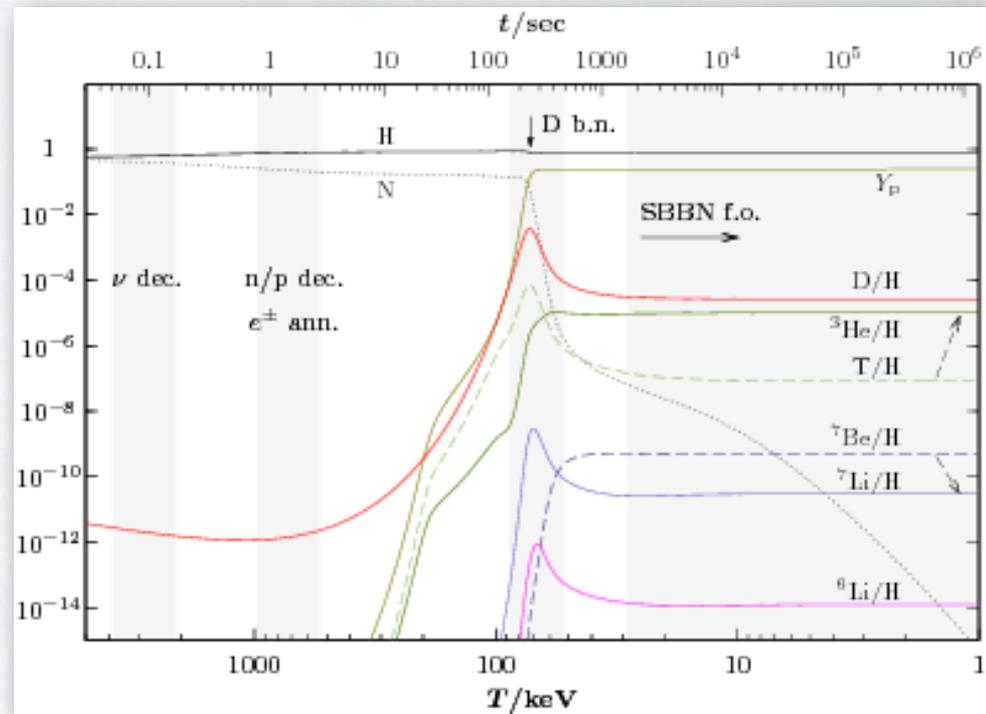
PLAN

- ▶ **Petit résumé de nucléosynthèse primordiale, introduction au “problème du lithium”**
- ▶ **Une possible solution nucléaire ? Réponse négative !**
- ▶ **Un regard de plus près à l’ astrophysique :
Mesure du Lithium “primordial”& processus d’altération possibles**
- ▶ **Solution exotiques en physique des particules : anciennes difficultés et dernières nouvelles**

BBN IN FOUR STEPS

- $T \gg 1 \text{ MeV}$: initial conditions dictated by NSE & input parameters.
- $T \sim 1 \text{ MeV}$: $p \leftrightarrow n$ freeze-out (weak physics... ^4He yield tracks n/p)
(departure from isospin equilibrium)
- $T \sim 0.1 \text{ MeV}$ Deuterium bottleneck opens (late, due to high entropy per baryon!)
- $0.1 \sim T \sim 0.01 \text{ MeV}$ nuclear reactions take place.
(departure from NSE equilibrium)

Despite availability of high-T,
BBN starts late and ends soon!
(inefficient combustion, leaving
fragile nuclear ashes behind)



A BBN SUMMARY

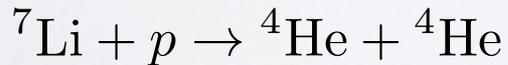
BBN is an overconstrained theory: all relevant observables depend only on the baryon to photon number density ratio η .

CMB provides an independent measurement of $\eta \sim 6 \times 10^{-10}$, hence BBN is parameter-free (a single nuclide determination suffices to test cosmology, wonderfully provided by D/H)

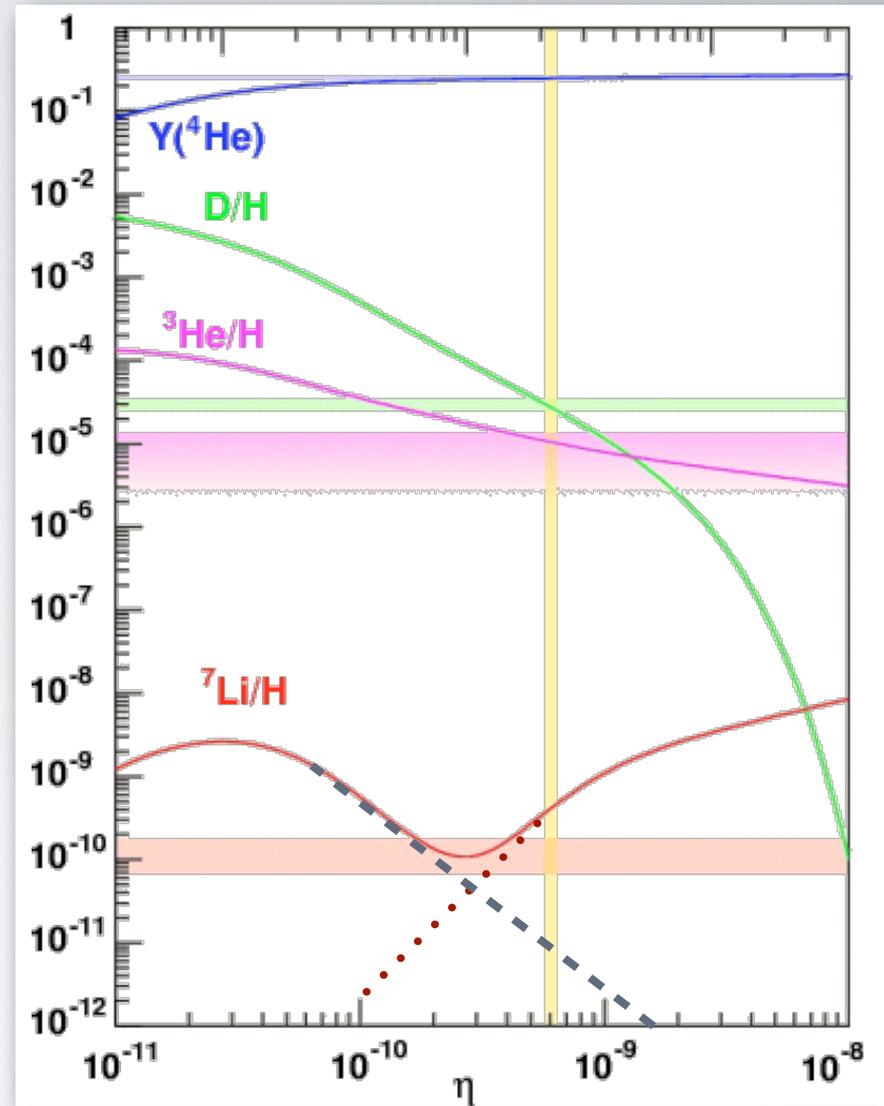
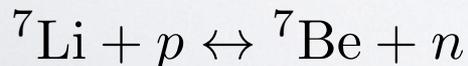
Only “disturbing feature”, ${}^7\text{Li}$ disagreement

Depending on the range of η , one of two reactions dominate production (turns out that we should be talking of a ${}^7\text{Be}$ problem!)

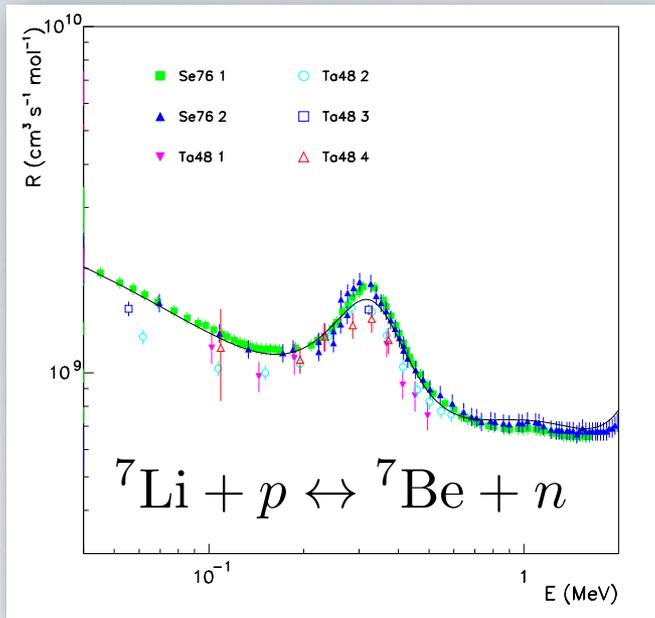
Destruction always dominated by



together with the isospin-equilibrium react.



RELEVANT REACTIONS WELL KNOWN



Over the past 10-15 years, the completeness and error budget have been extensively reviewed, by several independent groups. E.g.

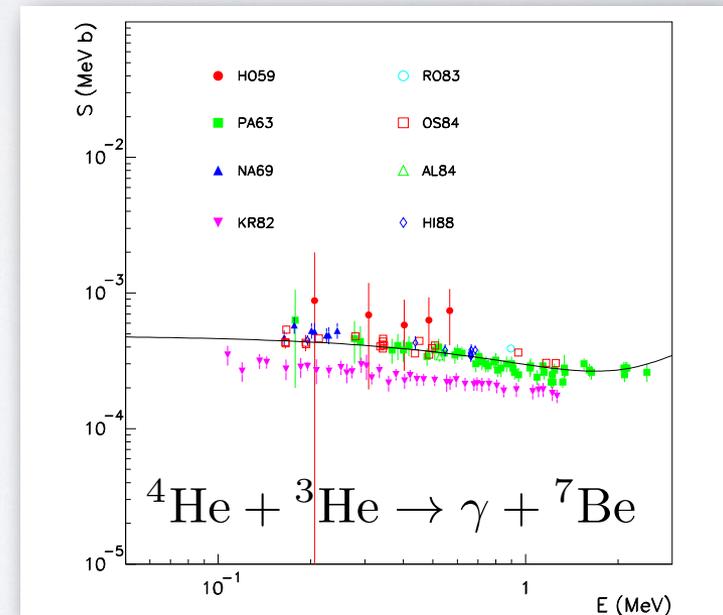
P. D. Serpico et al. "Nuclear reaction network for primordial nucleosynthesis: A detailed analysis of rates, uncertainties and light nuclei yields," JCAP 0412, 010 (2004)

Wrt these reaction, scaling established

$${}^7\text{Li}/\text{H} \sim 5.4 \times 10^{-10} R_{\tau\alpha}^{0.96} R_{\text{Lipn}}^{-0.71}$$

No room for errors of a factor 4 or more!

Are we missing some other relevant reaction?



...DOES NOT SEEM TO BE THE CASE

Even assuming order of magnitude uncertainty in poorly known secondary nuclear reactions, no room for nuclear resolution according to the analysis in

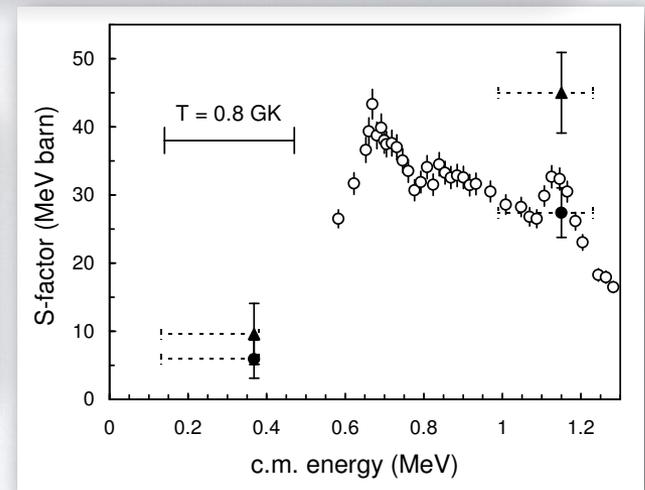
P. D. Serpico et al. "Nuclear reaction network for primordial nucleosynthesis: A Detailed analysis of rates, uncertainties and light nuclei yields," JCAP 0412, 010 (2004)

Some hope? 2 orders of magnitude uncertainty in poorly measured ${}^7\text{Be}(d,p){}^2{}^4\text{He}$ claimed a possible way out:

A. Coc et al. ApJ 600, 544 (2004)

eventually this possibility was experimentally ruled out

*C. Angulo et al.,
"The ${}^7\text{Be}(d,p){}^2{}^4\text{He}$ cross section at big bang energies and the primordial ${}^7\text{Li}$ abundances,"
ApJ 630, L105 (2005) [astro-ph/0508454].*



Solution either astrophysical, or due to new particle physics/cosmology

“PRIMORDIAL LITHIUM”?

Main problem

We cannot observe *primordial* abundances:
Stars easily burn Li, but other processes pre-galactic or galactic
(CR Spallation, ν process in SNaE, novae...) could have increased it



Observe systems with little chemical processing

Warm ($5700 \text{ K} < T < 6500 \text{ K}$) metal poor
dwarf stars in the halo

Correct for chemical evolution?

“metallicity plateau” found, means it’s
primordial? *Spite & Spite ‘82*

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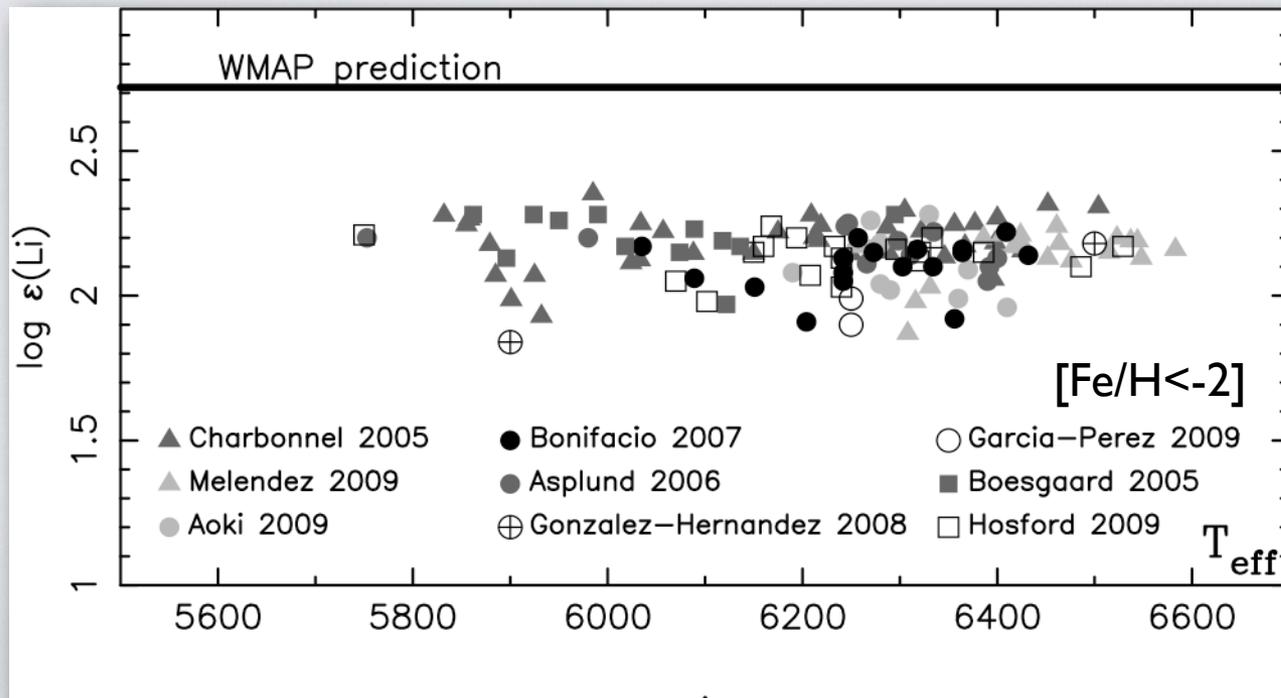


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A factor ~4
problem?

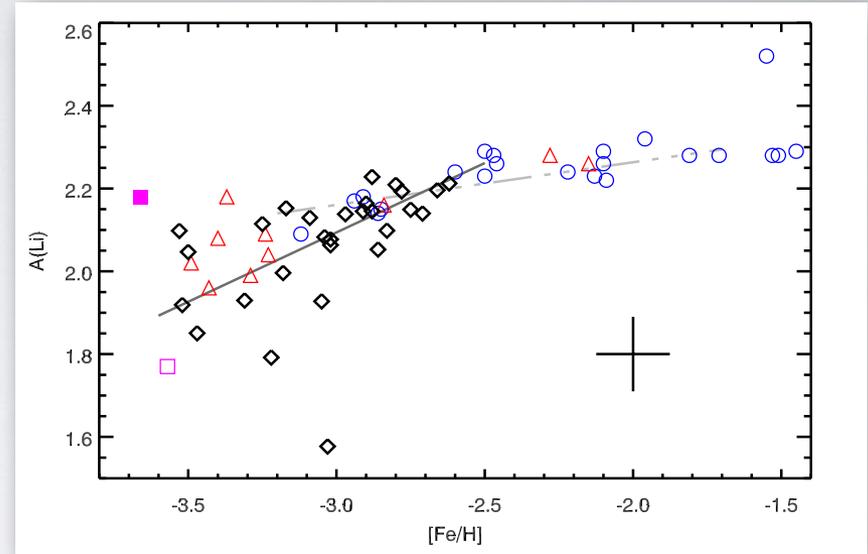
Caveat...

[Spite & Spite ‘10]

“PRIMORDIAL LITHIUM”? (CONT'D)

Anomalous dispersion around the average
& trend with metallicity established!!

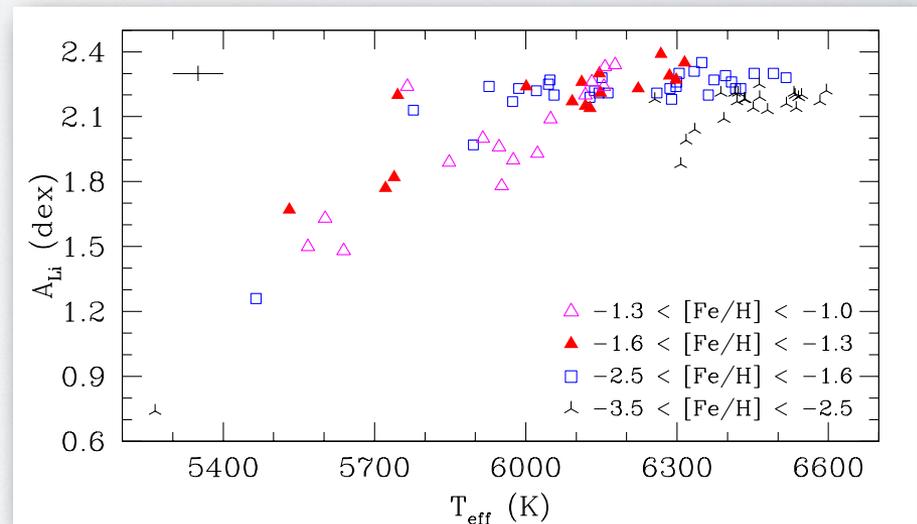
*e.g. Sbordone et al. A&A 522 (2010) A26
[arXiv:1003.4510]*



(cross is typical error size)

“Our results imply that the Li abundances
observed in Li plateau stars have been depleted
from their original values and therefore do not
represent the primordial Li abundance.”

*Melendez et al., A&A Letters, 515 (2010) L3
[arXiv:1005.2944]*



SO, WHY THE “PLATEAU”?

safe conclusion is that *some post-primordial processing is at play*; no simple astrophysical model universally accepted; it is still possible to interpret “the envelope” of the observed plateau values as primordial, which would be differ by a factor ~ 4 from predictions.

some hypotheses discussed

1. **MW Halo stars** did not form “unpolluted primordial gas” (Astration, Piau et al 2006).
Troubles with ω Cen, likely from captured dwarf, showing same Li (1008.1817)?
2. **Universal early or pre-galactic synthesis** (pop-III, flares, etc.)
Main issue is: why not other element ‘anomalies’? Energetics required? CS 22876 binary?
3. **in situ Depletion** (via diffusion/turbulence): appears likely but in most cases one has to fine tune “effective parameters” to get a “quasi-plateau”

A RECENT DEVELOPMENT

X. Fu, A. Bressan, P. Molaro, P. Marigo “Lithium evolution in metal-poor stars: from pre-main sequence to the Spite plateau” MNRAS 452, 3256–3265 (2015) [1506.05993]

propose the following possible scenario (sketch/personal summary):

- ◆ Primordial ${}^7\text{Li}$ all destroyed in early (pre-MS) phase, during ${}^2\text{H}$ burning.
- ◆ If stars form in an environment with residual gas available (*depends on metallicity range!*) accretion restores part of the ${}^7\text{Li}$
- ◆ Process stops when the star enters MS phase, its UV photons heat up the gas and wipe it away in a wind (*auto-regulation responsible for the plateau*)
- ◆ Too cool stars (=large convective region) destroy newly accreted one; warm ones (=thin convective region) preserve it (*T range where plateau observed*)

Promising, but yet to include more realistic effects like rotation, find “predictive” confirmation...

WHAT IF DUE TO PARTICLE PHYSICS?

Surprisingly enough, not easy to cook up a recipe to solve it! Why?

1. Solutions via annihilations/decays of exotic particle (cascades in the early universe) usually face the difficulty to alter *only* ${}^7\text{Li}$ while saving the agreement of other nuclei.
2. Those which manage to do so, usually require some mild “fine-tuning”, in order to work (usually dictated by nuclear physics, more later)
3. Even so, more often than not they violate other cosmological bounds (e.g. lead to too large CMB spectral distortions) and/or involve fine-tuned effects of multiple interactions (strong, e.m., weak).

Some recent developments (that solve at least 1 and 3, albeit not 2)

- Revisit the treatment of electromagnetic cascades leading to non-thermal BBN via photodisintegrations. Loophole found in the “standard physics” treatment, could resurrect solutions thought ineffective!
V. Poulin and PDS, Phys. Rev. Lett. 114, 9, 091101 (2015)
- Invoke one (or more) *light* particles, interacting via a *new force*
A. Goudelis, M. Pospelov and J. Pradler, Phys. Rev. Lett. 116, 21, 211303 (2016)

RECAP OF STANDARD LORE

*Basic processes (in a
high-entropy, radiation
dominated background)*

$$\gamma \gamma_{\text{th}} \rightarrow e^+ e^-$$

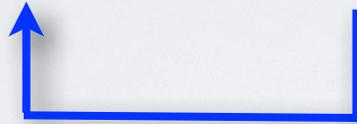
$$e \gamma_{\text{th}} \rightarrow e \gamma$$

RECAP OF STANDARD LORE

Basic processes (in a high-entropy, radiation dominated background)



Particle multiplication and energy redistribution

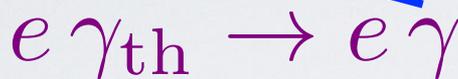


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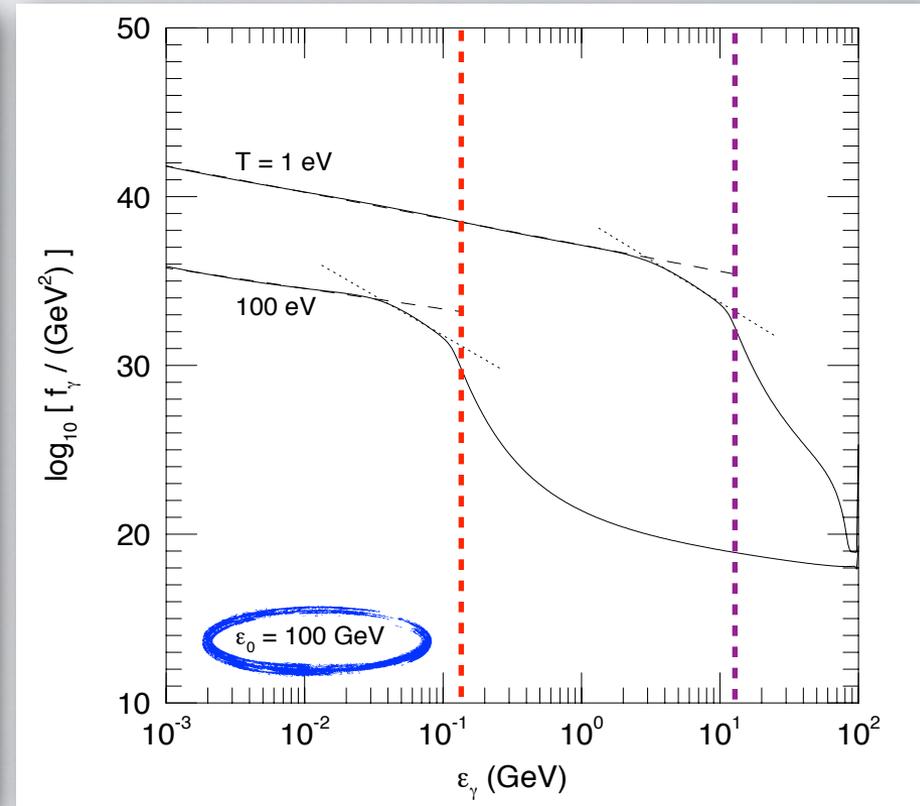
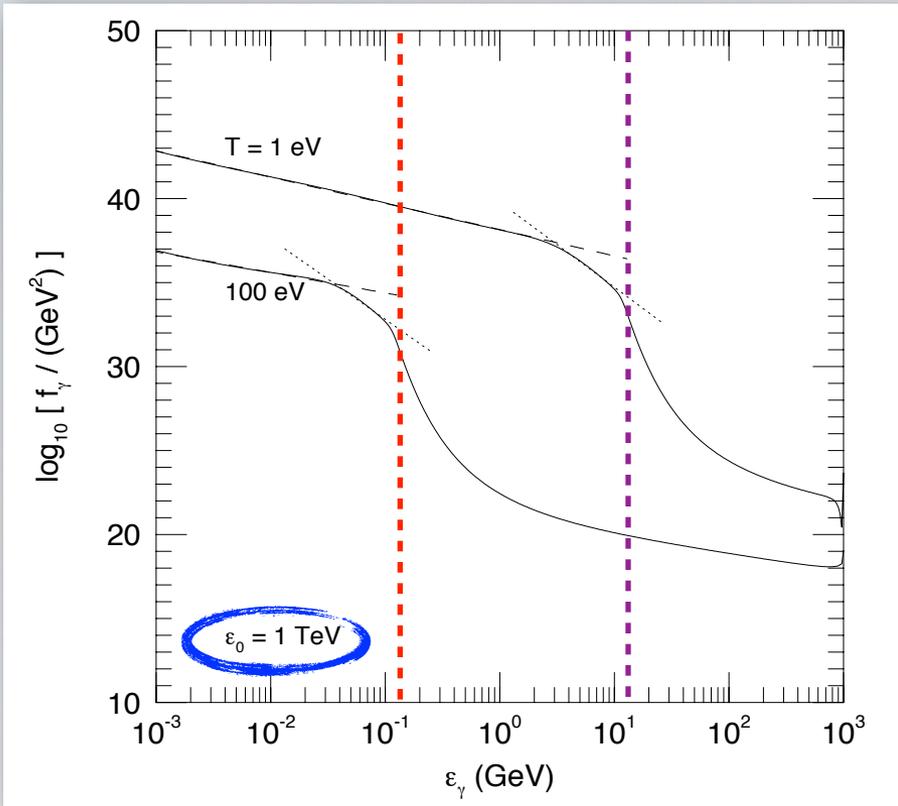
At threshold for P.P., $E_e \sim E_{\text{thresh}}/2$ and the corresponding maximal IC photon energy $E_X \sim E_{\text{thresh}}/3$

Below E_X , number of particles fixed by the number of e “available” (no more multiplication possible) and the Thomson-limit result $E_\gamma \propto E_e^2$ implies a scale-invariant spectrum goes as $E_\gamma^{-3/2}$

Above E_X and below the effective cutoff imposed by P.P., the **energy** of particles in the cascade is **conserved** ($E^2 dN/dE \sim \text{const}$), hence spectrum E^{-2}

On physical grounds we expect $E_{\text{cutoff}} \sim \# m_e^2/T$ Detailed simulations (for cosmo) yield $\# \sim 1/20$

EXAMPLES



$E_{\text{cutoff}}(100 \text{ eV}) \sim 120 \text{ MeV}$

$E_{\text{cutoff}}(1 \text{ eV}) \sim 12 \text{ GeV}$

& universal
(indep. of injected photon energy)

M. Kawasaki and T. Moroi,

“Electromagnetic cascade in the early universe and its application to the big bang nucleosynthesis,”

ApJ 452, 506 (1995) [astro-ph/9412055].

AN UNEXPLORED CORNER...

All cases simulated inject $E_\gamma \gg E_{\text{cutoff}}$
 But this is a **theoretical bias**
 (new physics must be “at high scale”)
 not a physical necessity!

$$E_{\text{cutoff}}(T=100 \text{ eV}) \sim 120 \text{ MeV}$$

$$E_{\text{cutoff}}(T=1 \text{ keV}) \sim 12 \text{ MeV}$$

M. Kawasaki and T. Moroi,
Apj 452, 506 (1995)
 [astro-ph/9412055]

Photo-disintegration energies E_{pd}
 for light nuclei range
 from $\sim 1.6 \text{ MeV}$ of ${}^7\text{Be}$ to
 $O(20) \text{ MeV}$ for ${}^4\text{He}$

What if $E_{\text{pd}} < E_\gamma < E_{\text{cutoff}}$, i.e. pair
 production is not operational but
 above threshold for photodisint.?
 Previous theory inapplicable!

This situation is physically possible at
 times after the end of standard BBN
 ($\sim 10 \text{ keV}$), which we focused on

$\epsilon_{\gamma 0} = 10 \text{ TeV}$

Temperature	P_{low}	$N_{\text{low}} \text{ GeV}^2$	P_{pp}	$N_{\text{pp}} \text{ GeV}^2$
1 eV	-1.57	1.6×10^8	-5.10	6.9×10^{-18}
10 eV	-1.34	5.4×10^8	-5.20	6.0×10^{-18}
100 eV	-1.22	1.7×10^9	-4.84	1.1×10^{-17}

$\epsilon_{\gamma 0} = 1 \text{ TeV}$

Temperature	P_{low}	$N_{\text{low}} \text{ GeV}^2$	P_{pp}	$N_{\text{pp}} \text{ GeV}^2$
1 eV	-1.56	1.4×10^8	-5.07	6.2×10^{-18}
10 eV	-1.34	4.9×10^8	-5.17	5.5×10^{-18}
100 eV	-1.22	1.4×10^9	-4.79	1.0×10^{-17}

$\epsilon_{\gamma 0} = 100 \text{ GeV}$

Temperature	P_{low}	$N_{\text{low}} \text{ GeV}^2$	P_{pp}	$N_{\text{pp}} \text{ GeV}^2$
1 eV	-1.56	1.4×10^8	-5.01	5.7×10^{-18}
10 eV	-1.33	4.7×10^8	-5.15	5.3×10^{-18}
100 eV	-1.22	1.3×10^9	-4.74	1.1×10^{-17}

$\epsilon_{\gamma 0} = 10 \text{ GeV}$

Temperature	P_{low}	$N_{\text{low}} \text{ GeV}^2$	P_{pp}	$N_{\text{pp}} \text{ GeV}^2$
1 eV	—	—	—	—
10 eV	-1.33	4.5×10^8	-5.12	5.5×10^{-18}
100 eV	-1.22	1.3×10^9	-4.77	9.6×10^{-18}

SOLUTION FOR THE NEW REGIME

Need to account for remaining processes kinematically allowed



Assuming all interactions are catastrophic, the relevant Boltzmann equation writes

$$\frac{\partial f_{\gamma}(E_{\gamma})}{\partial t} \simeq -\Gamma_{\gamma}(E_{\gamma}, T(t)) f_{\gamma}(E_{\gamma}, T(t)) + \mathcal{S}(E_{\gamma}, t)$$

whose stationary solution is

$$f_{\gamma}^{\text{S}}(E_{\gamma}, t) = \frac{\mathcal{S}(E_{\gamma}, t)}{\Gamma_{\gamma}(E_{\gamma}, t)}$$

(Hubble expansion is much slower than all these particle physics interaction rates, but can be accounted for)

where for a decaying particle

$$\mathcal{S}(E_{\gamma}, t) = \frac{n_{\gamma}^0 \zeta_X (1 + z(t))^3 e^{-t/\tau_X}}{E_0 \tau_X} p_{\gamma}(E_{\gamma}, t)$$

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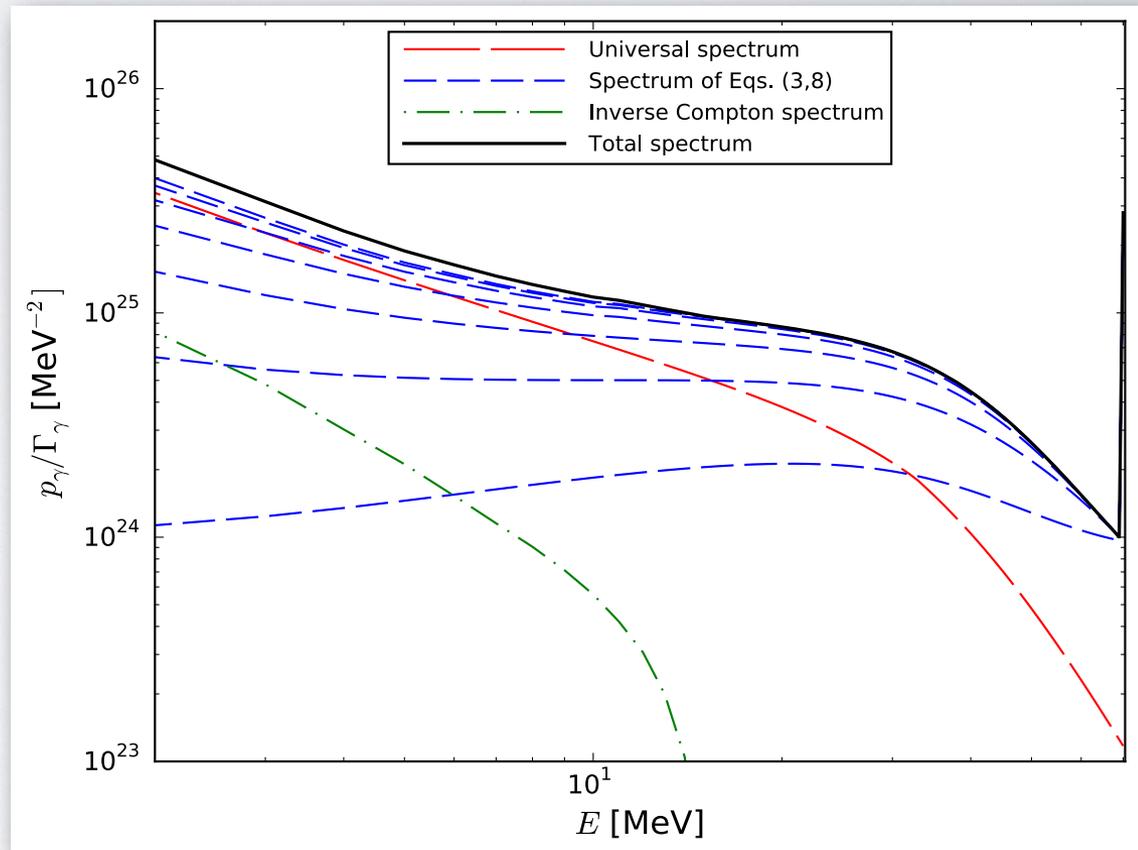
ITERATIVE SOLUTION

Exact at high-E, need to account for re-injection for the lower energy

$$\mathcal{S}(E_\gamma, t) \rightarrow \mathcal{S}(E_\gamma, t) + \int_{E_\gamma}^{\infty} dx K_\gamma(E_\gamma, x, t) f_\gamma(x, t)$$

► At lower energies, also some effect due to up-scattered thermal photons by non-thermal electrons produced by 1st-generation photons (“tertiary” component)

► The method converges quickly (<10% errors with 4 iterations) and can be generalized to fully account for the coupled eq. for the electrons



PROOF OF PRINCIPLE

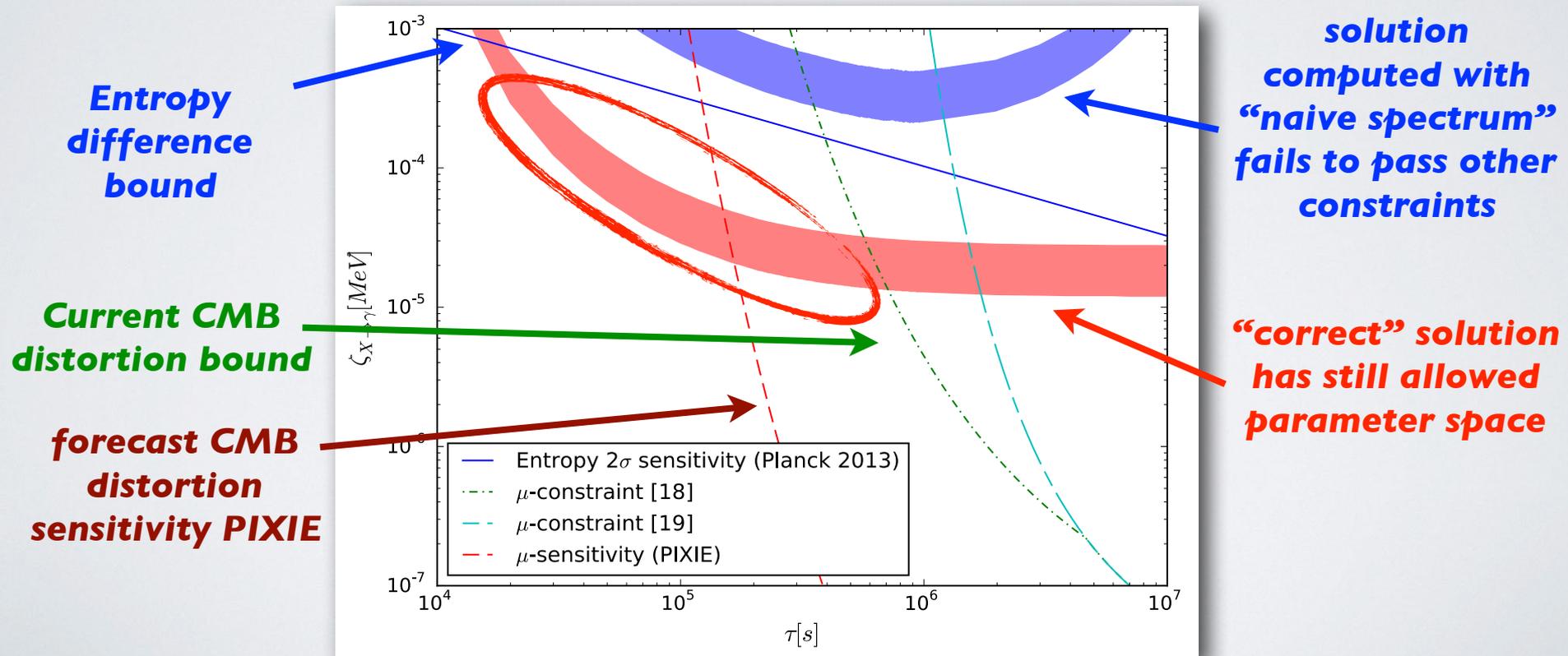
- ▶ Inject γ 's with $E > 1.6$ MeV (${}^7\text{Be}$ photodis. threshold, ${}^7\text{Be}$ =dominant progenitor of ${}^7\text{Li}$ into which it eventually decays) & $E < 2.2$ MeV (${}^2\text{H}$ photodis. threshold, next to most fragile nuclide)
- ▶ By construction, this does not perturb all other nuclear yields, one can adjust normalization to deplete by a factor 3-4. The solution is analytical:

$$\ln \left(\frac{Y_{7\text{Be}}(z_i)}{Y_{7\text{Be}}(z_f)} \right) = \int_{z_f}^{z_i} dz' \frac{n_{\gamma}^0 \zeta_X \sigma_{*}(E_0) c e^{\frac{-1}{2H_r^0 \tau_X (z'+1)^2}}}{E_0 H_r^0 \tau_X \Gamma(E_0, z')}$$

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ACTUAL (SIMPLE) MODEL

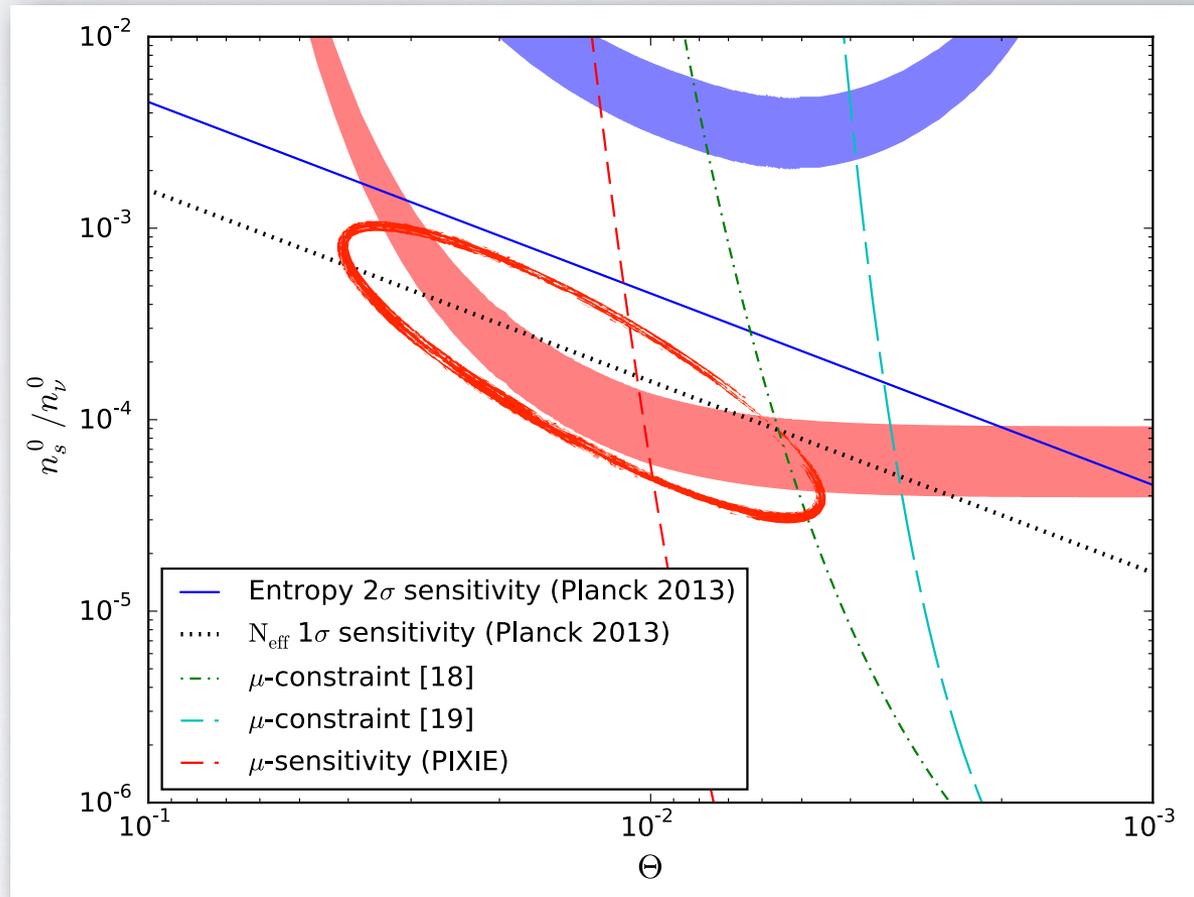
Maybe proof of principle too idealized, can it be realized in actual models? It can, for parameters that pass both cosmological and laboratory bounds!

Below, results for sterile ν , ~ 4 MeV mass ($\pm 20\%$) mixing mostly with ν_μ and/or ν_τ
Branching ratios for decay $\sim 1:0.1:0.01$ in $3\nu : \nu e^+e^- : \nu\gamma$

If using “universal” spectrum,
this type of simple models
known to fail, see

*H. Ishida, M. Kusakabe and H. Okada,
PRD 90, 8, 083519 (2014)*

**Obviously, we do not
claim that this is the solution.
We just stress that relatively
simple models (naively
discarded as badly ineffective)
can do the job**



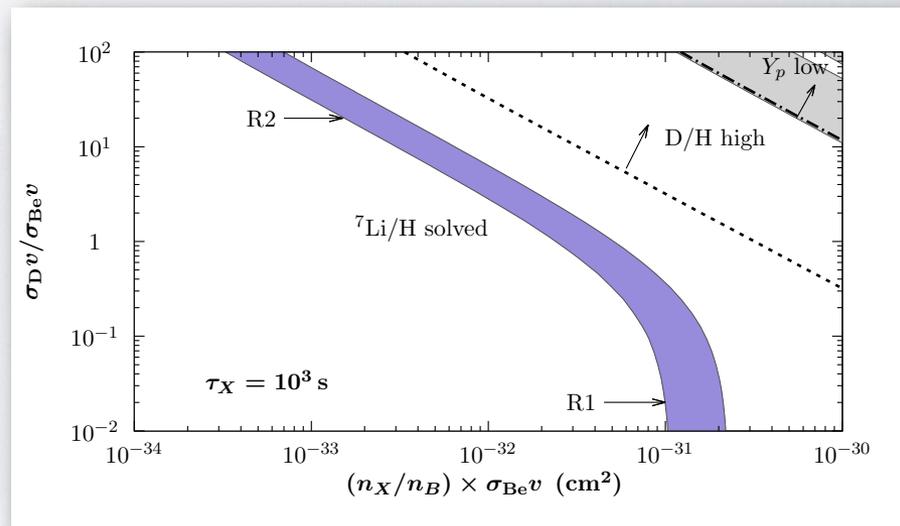
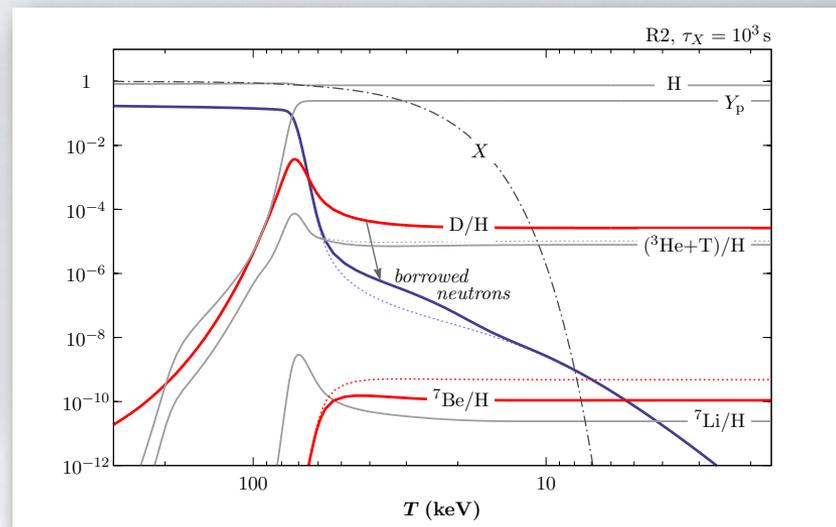
NEW LIGHT PARTICLES & NEW FORCES?

New metastable particle, X , with lifetime $\sim 10^3$ s, sufficiently light not to disrupt He ($m_X \sim 5$ MeV), sufficiently abundant/interacting with ${}^7\text{Be}$ and D via



can do the job in a sufficiently wide parameter space!

Key trick: you allow D destruction at the same time, but provided it stops by $t \sim 10^3$ s (X decay), neutrons are recaptured, and reform D!



$$\mathcal{L}_{aq} = \frac{\partial_\mu a}{f_d} \bar{d} \gamma_\mu \gamma_5 d \quad \Rightarrow$$

$$\mathcal{L}_{a\pi N} = \frac{\partial_\mu a}{f_d} \left[f_\pi \partial_\mu \pi^0 + \frac{4}{3} \bar{n} \gamma_\mu \gamma_5 n - \frac{1}{3} \bar{p} \gamma_\mu \gamma_5 p \right]$$

Actual model: e.g. axion-like particle coupled mostly with d-quarks in a multiple Higgs scenario ($f_d \sim \text{TeV}$ scale)

CONCLUSIONS

- ▶ The long-standing *Lithium problem* appears to be due either to astrophysical or particle physics/cosmological causes, not to lack of nuclear physics data/theory.
- ▶ In the former possibility, the goal is to have *testable* models that possibly offer dynamical arguments for the Spite & Spite plateau.
- ▶ Particle physics solutions have not come easy, either. New developments seem to offer unexpected hope either from better treatment of standard model processes at *low scales* or from new physics (with *light* degrees of freedom).
- ▶ There may be room for an interplay with nuclear physics. E.g.
 - in *V. Poulin, PDS, Phys. Rev. D 91, 10, 103007 (2015)*, we pointed out possible need for photo disintegration data in the ~ 100 MeV range, to compute realistic BBN bounds on cascades in the early universe.
 - In *A. Goudelis, M. Pospelov, J. Pradler, Phys. Rev. Lett. 116, 21, 211303 (2016)* simple recipe to compute relevant “exotic” nuclear reactions, such as:
How well does this approximation work?
$$\frac{\sigma_{\text{abs},i\nu}}{\sigma_{\text{photo},iC}} \simeq \frac{C_i}{4\pi\alpha} \times \frac{m_a^2}{f_d^2}$$

spin factors 

Merci pour votre attention!